DBJ-003-1015001 Seat No. ________

B. Sc. (Sem. V) (W.E.F.-2018) Examination

June – 2022

Mathematics : MATH-05A

(Mathematical Analysis-1 and Abstract Algebra-1)

Faculty Code : 003

Subject Code : 1015001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) Attempt any five questions.

(2) Figures to the right indicate full marks of the question.

1 (a) Answer the following questions : **4**

(1) Define Interior point.

(2) Define Dense set.

(3) If $X = [0,2), d$ is discrete metric on X.

Then $N\left(0, \frac{1}{2}\right)$ = _____.

(4) Every metric space is pseudo metric space. (True/False)

(b) Show that every singleton set in $\mathbb{R}$ is with usual metric is not open. **2**

(c) Prove that union of arbitrary family of open set is open in any metric space. **3**

(d) Let A and B be any two subsets of a metric space (X, d). Then prove that **5**

(i) $(A \cup B)' = A' \cup B'$.

(ii) A is open if and only if $A = A^{\circ}$.

2 (a) Answer the following questions : **4**

(1) Define Exterior point.

(2) Define Boundary point.

(3) If X = [0,2), d is usual metric on X.

Then $N\left(0, \frac{1}{2}\right)$ = _____.

(4) Every finite set in any discrete metric space is open. (True/False)

(b) Prove that $\frac{1}{3}$ is in cantor set. **2**

(c) Let (X, d) be a metric space. Then prove that every distinct points in X have disjoint neighborhoods. **3**

(d) Let (X,d) be a metric space and $d_1 : X \times X \to \mathbb{R}$ be a function defined as $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}; \forall x,y \in X$. Prove that d_1 is metric on X. **5**

3 (a) Answer the following questions : **4**

(1) If $P = \left\{0, \frac{1}{3}, \frac{1}{2}, 1\right\}$ is a partition of [0,1] then find $||P||$ is ________.

(2) Every continuous function is integrable. (True/False)

(3) Define : Riemann integration.

(4) Define : Finner partition.

(b) Find $L(P.f)$ and $U(P,f)$ for the function **2**

$f(x)=x^2, x\in[0,1]$ and $P=\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1\right\}$.

(c) If $f\in R_{[a,b]}$ then prove that $f^2\in R_{[a,b]}$. **3**

(d) State and prove necessary and sufficient condition for a bounded function on $[a, b]$ to be R-integrable. **5**

4 (a) Answer the following questions : **4**

(1) If $f(x)=x, x\in[0,1]$ and $P=\left\{0,\frac{1}{2},1\right\}$ then find $U(P,f)$.

(2) Define : Oscillation of f in $[a, b]$.

(3) Define : Norm of a partition.

(4) Every monotonically decreasing function is integrable. (True/False)

(b) Prove that $\underline{\int_a^b} f dx \le \overline{\int_a^b} f dx$; where f is real valued function defined on [a,b]. **2**

(c) Show that a constant function $f(x)=k$ is integrable and also prove that $\int_a^b k dx = k(b-a)$. **3**

(d) If $f\in R_{[a,b]}$ and $g\in R_{[a,b]}$. Then prove that $f.g\in R_{[a,b]}$. **5**

5 (a) Answer the following questions : **4**

(1) Define : Binary operation.

(2) State Second Mean Value Theorem {Weierstrass form}.

(3) Convert $\lim\limits_{x\to\infty}\frac{1}{n}\left[\frac{1}{n}+\frac{2}{n}+....+\frac{n}{n}\right]$ as definite integral.

(4) Consider $\mathbb{Z}_7$ under multiplication modulo 7 then find order of 3.

(b) Prove that a group G is commutative if $a^2 = e, \forall a \in G$. **2**

(c) Show that the subset $S = \{a + b\sqrt{5} \mid a,b \in \mathbb{Q}, a^2 + b^2 \neq 0\}$ of $\mathbb{R}$ is a group under usual multiplication of two real numbers. **3**

(d) For 0 < a < b, Prove that **5**

$$\frac{\pi^2}{2b} \leq \int_0^{\pi} \frac{x}{a\cos^2\frac{x}{2} + b\sin^2\frac{x}{2}}dx \leq \frac{\pi^2}{2a}.$$

6 (a) Answer the following questions : **4**

(1) State Second Mean Value Theorem {Bonnett's form}.

(2) Define General Linear Group.

(3) Define Semi Group.

(4) Consider $\mathbb{Z}_{10}$ under addition modulo 10 then find order of 2 and 7.

(b) What is the primitive of $f(x) = \log x$ in [1,2] ? **2**

(c) Show that set of all non-zero real numbers with operation $a * b = \frac{ab}{3}$ is an abelian group. **3**

(d) State and prove First Mean Value Theorem for integral calculus. **5**

7 (a) Answer the following questions : **4**

(1) Define : Center of a group.

(2) All generators of Z_{20} are prime numbers. (True/False)

(3) Define : Even permutation.

(4) Define : Left cosets.

(b) Show that union of two subgroups need not be a subgroup. **2**

(c) Let G be an abelian group under multiplication with identity e. Then show that $H = \{x^2 \mid x \in G\}$ is a subgroup of G. **3**

(d) Each Permutation $f \in S_n$ can be expressed as a composition of disjoint cycles. **5**

8 (a) Answer the following questions : **4**

(1) Define : Euler phi function.

(2) Define : Index of a subgroup.

(3) The symmetric group S_{10} has 10 elements. (True/False)

(4) Every cyclic group is abelian (True/False)

(b) Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 7 & 1 & 4 & 6 & 5 \end{pmatrix}$ and **2**

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 3 & 7 & 6 & 5 & 4 & 2 \end{pmatrix}.$$

Compute f^{-1} and g^{-1}.

(c) Let H be a subgroup of G and let a and b belong to G. Then prove that $aH = bH$ if and only if $a^{-1}b \in H$. **3**

(d) State and prove Lagranges Theorem. **5**

9 (a) Answer the following questions : **4**

(1) An abelian group can't be isomorphic to a nonabelian group. (True/False)

(2) Z/nZ is cyclic of order n. (True/False)

(3) Define : Normal subgroup.

(4) Define : Inner automorphism.

(b) Let $\phi:(G,*) \to (G',*')$ be an isomorphism. then prove that $\phi(a^{-1}) = [\phi(a)]^{-1}, \forall a \in G$. **2**

(c) Prove that the relation $\cong$ isomorphism is symmetric. **3**

(d) Let $(G,*)$ be a group and H is a subgroup of G. Then prove that H is normal subgroup if and only if $(Ha)(Hb) = Hab, \forall a,b \in G$. **5**

10 (a) Answer the following questions : **4**

(1) A_n is a normal subgroup of S_n. (True/False)

(2) Every one-to-one function between groups is an isomorphism. (True/False)

(3) Define : Factor group.

(4) Define : Simple group.

(b) Let $\phi : (G, *) \to (G', *')$ be an isomorphism. Then prove that $\phi(e) = e'$; where e and e' are identity element of G and G' respectively. **2**

(c) Let $(G, *)$ be a group and $H = \{a^2 \mid a \in G\}$ is a subgroup of G. Then prove that H is normal subgroup of G. **3**

(d) State and prove Cayley's theorem. **5**